**Relativistic Quantum Mechanics**

**Hilbert Space**

So now our wavefunction exists in an enlarged HS:



where ms spans 4 dimensional column vector space (spinor space), instead of the previous 2 dimensional space it used to reside in. So we can say,



We’ll typically just write this as:



where **χ** and **Φ** are both 2D column vectors. And we’ll notate:



for short. This notation is handy because the Dirac equation is block diagonal/off diagonal, and so doesn’t split up the components of **χ** and **Φ**. A general wavefunction will now look like,



**Representation of Observables**

So the usual real space observables like r, p, L, H, etc. don’t change. And they would just be written as usual, multiplied by a 4D unit matrix in our augmented HS. The spin operator already operates in half the space, and now we’d just carry it over to the other half. We’d say:



where **σ** is the 2D vector Pauli-spin matrix. As noted above, we have two new operators , and . And their explicit representation in this 4D space is:



These are Hermitian. I wonder if they correspond to any particular observable? And we have the related γ operators,



**Time-Development**

We must still have:



**Some nice operator identities**

Before stopping for the moment, let’s consider a useful identity. Let M and N be vector operators. Then consider the following construction:



So,



And consequently



or for short, leaving out the implicit identity matrix,



Note it follows in particular that:



Can also go to higher order products,



Then use,



to write,



And so we have:



And with that formula, we could continue to reduce any product of σ’s back down to some linear combination. The γμ also have worthwhile properties to investigate. Consider the Weyl representation. It’s often written in shorthand as:



where **1** is the identity matrix, not a vector. Might note that:



and in particular,



Now let’s look at anti/commutation relations. First the Dirac representation:



and also have:



The anticommutation relations are generally written succinctly as:



Apropos the Weyl representation we have:



and also have:



And we can say, like before,



All the relations are the same, w/r to themselves, as we’d expect since they’re related via unitary transformation. Here’s some more:



and being overly meticulous,



which implies,



where **1** is a 4×4 identity matrix. Now just want to look at the rotation matrix, in spin basis. First, using:



we have that:



etc., and so:



and we get:



This sort of construction will be useful when we consider Lorentz boosts to spinors.